62-1571

THE MIXING OF JETS IN A CHANNEL WITH VARIABLE CROSS-SECTION

By

O. V. Yakovlevskiy

UNEDITED ROUGH DRAFT TRANSLATION

THE MIXING OF JETS IN A CHANNEL WITH
VARIABLE CROSS-SECTION

By: O. V. Yakovlevskiy (Moscow)

English Pages: 13

Source: Izvestiya Akademii Nauk SSSR, OTN, Mekhanika i Mashinostroyeniye, No. 1, 1962, pp. 66-72.

SC-1612
SOV/179-62-0-1-8/27

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.	PREPARED BY: TRANSLATION SERVICES BRANCH FOREIGN TECHNOLOGY DIVISION WP-AFB, OHIO.

P

THE MIXING OF JETS IN A CHANNEL WITH VARIABLE CROSS-SECTION

O. V. Yakovlevskiy

In gas machines we encounter currents with non-uniform profiles of gas-dynamic parameters, e.g., velocity, this non-uniformity gradually smoothing out downstream. Similar flows are observed in particular in the mixing chambers of ejectors, two-circuit jet engines [1], etc. In some practical cases these flows occur in channels, the form of which differs from the cylindrical, i.e., in channels with variable cross-section.

In planning such gas machines and apparatus it is necessary to know the laws of change in the non-uniformity of the stream along the flow-through section. The methods of calculating the parameters of a non-uniform stream in a channel of arbitrary shape have not, however, yet been established. There exists only the monograph [2] of A. Ya. Cherkez on a theory devoted to flow in the cylindrical blending chamber of an ejector.

In the present work an attempt is made to devise a method of calculating the flow of a fluid with non-uniform velocity distribution of the jet type in a channel with variable cross-section.

<u>1. Initial premises and equations</u>. When mixing coaxial streams in a channel (Fig. 1) it is, as is well known [2], possible to separate at least two essentially different regions of flow. In the first of these, called the primary zone, the dimensions of the region of

flow mixture are less than the transverse dimensions of the channel; whereas in the following secondary zone the mixing process already embraces the whole cross-section of the stream. The flow in precisely this zone will be examined below.

From analysis of the velocity profiles in the second zone of the stream in cylindrical and conical (effuser) channels the conclusion has been successfully drawn that the the velocity distributions in different cross-sections reduced to dimensionless form by two parameters maintain their similarity along the channel. Concretely, it is question of autosimulation of the profile of dimensionless velocity of the form

$$\delta u^{\circ} = \frac{u-g}{u_m-g} = \psi\left(\frac{y}{R}\right) \tag{1.1}$$

Here u is the longitudinal component of flow velocity; u_m, velocity on the x axis; g, average (areal) flow velocity in the given section; R, present radius of the channel; and y, distance from the axis.

The experimental data of A. Ya. Cherkez [2] and K. Viktorin [3] for a cylindrical channel are shown in Fig. 2 treated as the function $\delta u^{\circ}(\xi)$, where $\xi = y/R$; and the results of the author's experiments for a mixing chamber of the effuser type (truncated cone with aperture angle of 8°20'), in Fig. 3. As we see, the profile of dimensionless velocity $\delta u^{\circ}(\xi)$ is similar in both cases in the different cross-sections of the stream.

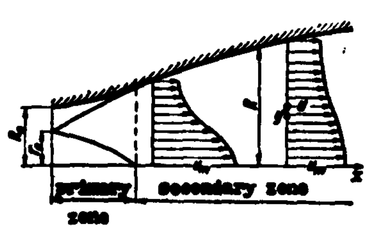

Fig. 1. Diagram of jet propagation in a channel with variable cross-section.

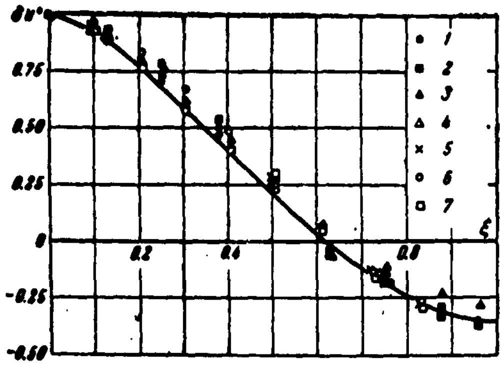

Fig. 2. Profile of relative excess velocity $\delta u^{0}(\xi)$ in a cylindrical channel. Curve from Formula (1.2); points from experiments (dark-air ejector [2], light-water mixer [3]) for following distances from entry cross-section relative to channel radius: 1) 7.5, 2) 9.5, 3) 13.5, 4) 7.3, 5) 8.3, 6) 9.4, 7) 10.4.

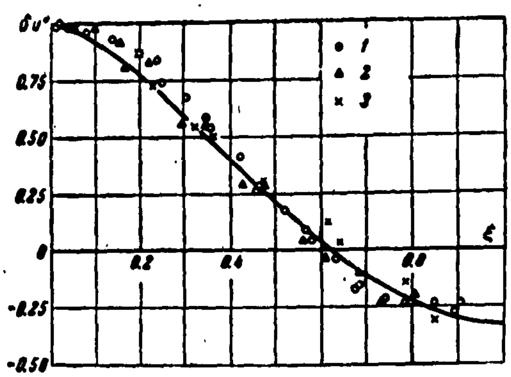

Fig. 3. Profile of relative excess velocity $\delta u^{0}(\xi)$ in effuser channel ($\alpha = -0.0721$). Curve from Formula (1.2); points from author's experiments at following distances from entry cross-section: 1) 462 mm, 2) 662 mm, 3) 746 mm.

By analogy with the free turbulent stream let us assume that autosimulating distribution of dimensionless velocity in the cross-section of the jet stream in the channel is described by the so-called Schlichting law which is written

$$\delta u^{\circ} = \frac{\varphi(\xi) - A(1)}{1 - A(1)}, \qquad \varphi(\xi) = \left(1 - \xi^{\frac{3}{2}}\right)^{2} \tag{1.2}$$

when applied to the case under examination.

Coefficient A(1) is a numerical constant which is selected so that the equation of discontinuity will be identically satisfied when using profile (1.2).

The curves corresponding to relationship (1.2) are also shown in Figs. 2 and 3; it is evident that in their first approximation they describe the autosimulating profile of velocity $\delta u^{0}(\xi)$ rather well.

Thus, one of the initial prerequisites of the calculation method is the similarity in the profiles of dimensionless velocity which was

discovered on analyzing the experimental data; in this, the autosimulating distribution of velocity $\delta u^{o}(\xi)$ may be described by relationship (1.2).

Moreover, the following assumptions were made in the calculations: the fluid is considered incompressible (ρ = const), the static pressure p is assumed to be constant at every cross-section, and the force of friction between the fluid and the wall of the channel is neglected. The last assumption is justified in that the extent of the channel in which mixture takes place is usually small and amounts to only a few diameters of the channel.

Consequently the problem is reduced to determining the dependence of three unknown values, u_m, g, and p, on longitudinal coordinate x. Axisymmetrical fluid flow in the examined case is, as is well known [4], determined by the equations

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp}{dx} - \frac{1}{y}\frac{\partial \tau_T y}{\partial y}, \qquad \frac{\partial \rho u}{\partial x} + \frac{1}{y}\frac{\partial \rho v y}{\partial y} = 0 \tag{1.3}$$

Here u and v are the components of velocity of average motion; and the stress of turbulent friction τ_t according to L. Prandtl is described by the formula

$$\tau_T = \rho l^2 \frac{\partial u}{\partial y}\left|\frac{\partial u}{\partial y}\right|$$

System (1.3) may be easily converted to the form

$$\frac{\partial}{\partial x}(\rho u^2 y) + \frac{\partial}{\partial y}(\rho u v y) = -y\frac{dp}{dx} - \frac{\partial \tau_T y}{\partial y}, \qquad \rho v y = -\frac{\partial}{\partial x}\int_0^y \rho u y\, dy \tag{1.4}$$

If both halves of the first of the Equations (1.4) are multiplied by dy and integrated from 0 to an arbitrary y and then the value $\rho v y$ from the second of these Equations (1.4) is substituted, we get

$$\frac{d}{dx}\int_0^y \rho u^2 y\, dy - u\frac{d}{dx}\int_0^y \rho u y\, dy = -\frac{y^2}{2}\frac{dp}{dx} - \tau_T y \tag{1.5}$$

This is the first of the correlations of the system of equations for determining the above-listed three variables. The second

correlation is derived from (1.5) if the upper limit in the integrals entering here is set equal to channel radius R and the boundary condition at the wall, y = R, $\tau_t = 0$, is satisfied:

$$\frac{dp}{dx} = -\frac{2}{R^2}\frac{d}{dx}\int_0^R \rho u^2 y dy \tag{1.6}$$

The third correlation in our system is the condition for maintaining mass in the channel

$$\frac{d}{dx}\int_0^R \rho u y dy = 0, \text{ or } G = \text{const} \tag{1.7}$$

and serves to determine the average (areal) stream velocity in each cross-section.

Let us introduce the dimensionless coordinates and dimensionless velocities

$$x^\circ = \frac{x}{R_0}, \quad \xi = \frac{y}{R}, \quad R^\circ = \frac{R}{R_0}, \quad u^\circ = \frac{u}{g}, \quad u^\circ_m = \frac{u_m}{g}, \quad U = \frac{u^\circ_m - 1}{1 - A(1)} \tag{1.8}$$

where R is the radius of the channel at some fixed cross-section.

Let us replace the gradient of pressure in Eq. (1.5) by its expression in (1.6); in doing so we will assume that the relationship of mixture path l to the channel radius in the same section remains constant along the channel and can depend only on ξ, i.e., $l/R = \varkappa(\xi)$; in addition, we will use expression (1.2) to distribute velocity u in the cross-section of the stream.

As a result we obtain the following differential equation for determining the dimensionless velocity on channel axis U

$$R^\circ [M(\xi) + N(\xi) U] \frac{dU}{dx^\circ} = \mp \varkappa^2 T(\xi) U^2 + 2\frac{dR^\circ}{dx^\circ} U [2M(\xi) + P(\xi) U] \tag{1.9}$$

Here the top sign before the first member on the right side is taken if the velocity on the flow axis is greater than at the wall; and the bottom sign, in the opposite case. Moreover, in Eq. (1.9) the following designations are introduced:

$$
\begin{gathered}
M(\xi) = A(\xi) - \xi^2 A(1), \qquad T(\xi) = 18\xi^2\varphi(\xi) \\
N(\xi) = 2[B(\xi) - \xi^2 B(1)] - M(\xi)[3A(1) + \varphi(\xi)] \\
P(\xi) = B(\xi) - \xi^2 B(1) - 2A(1)M(\xi)
\end{gathered} \tag{1.10}
$$

$$
A(\xi) = 2\int_0^\xi \varphi(\xi)\,\xi\,d\xi, \quad A(1) = 0.2571, \quad B(\xi) = 2\int_0^\xi \varphi^2(\xi)\,\xi\,d\xi, \quad B(1) = 0.1335
$$

If function $\varphi(\xi)$ were an autosimulating solution of the motion equations Eq. (1.9) would not contain ξ, that is, the coefficients of $M(\xi)$, $N(\xi)$, $T(\xi)$, and $P(\xi)$ would be proportional to each other. But in our case these coefficients possess no exact proportionality; one can convince himself of this from an examination of Fig. 4 which lists the relationships $M(\xi)$, $N(\xi)$, and $P(\xi)$. In this we do not examine the change in value of $T(\xi)$ since it has in principle no significance as it is the co-factor of the empirical constant $\varkappa^2$. In order to simplify the subsequent analysis we will relate the coefficients of Eq. (1.9) to the value of $M(\xi)$ and introduce the following designations for these relative coefficients:

$$
a(\xi) = \frac{N(\xi)}{M(\xi)}, \qquad b(\xi) = \frac{T(\xi)}{M(\xi)}, \qquad c(\xi) = \frac{P(\xi)}{M(\xi)}
$$

Substituting these coefficients in Eq. (1.9) and also introducing the value $k = \varkappa^2 b(\xi)$ we finally get

$$
R^\circ(1 + aU)\frac{dU}{dx^\circ} = \mp kU^2 + 2\frac{dR^\circ}{dx^\circ}U(2 + cU) \tag{1.11}
$$

Since there are no objective criteria for the selection of the value of ξ at which the coefficients of $a(\xi)$, $b(\xi)$ and $c(\xi)$ should be figured, we will, in our desire to avoid arbitrariness, find their average integral values in accordance with the formulas

$$
a_{av} = 2\int_0^1 a(\xi)\,\xi\,d\xi \qquad \text{etc.}
$$

Completing the integration we finally get

$$
\begin{gathered}
a_{av} = 0.3460, \qquad b_{av} = 15.0, \\
c_{av} = 0.1664
\end{gathered} \tag{1.12}
$$

We will use these values of the coefficients of Eq. (1.9) in what follows.

2. Analysis of the solution. In the general case when the relationship $R^o(x^o)$ has an arbitrary form, Eq. (1.11) is an Abelian equation of the second sort [5] and its solution is not expressed in quadratures. In two frequent cases which are examined below, the solution of differential Eq. (1.11) may be successfully expressed through elementary functions.

Case 1. A cylindrical channel (R^o = const = 1). In the case of a cylindrical channel Eq. (1.11) is greatly simplified and its solution in the boundary condition where $x^o = 0$, $U = U_*$ has the form

$$\frac{1}{U} - \frac{1}{U_*} - a \ln \frac{U}{U_*} = \pm k x^o$$

If we substitute here the numerical value of coefficient a according to Formula (1.12) and relate the excess velocity on the stream axis U to its value in the first section U_*, i.e., introduce $U^o = U/U_*$, we finally get

$$\frac{1}{U^o} - 1 - 0.80\, U_* \lg U^o = \pm k U_* x^o \qquad (2.1)$$

The empirical constant k in this equation is approximately 0.26 according to estimates based on the experimental data of different authors [2, 3, 6]. The change in the static pressure along the channel is found from Eq. (1.6). The change in static pressure is related to velocity pressure corresponding to the average (areal) velocity g, which in the case examined remains constant along the channel (g = const = g_*), and is determined from the formula

$$\Delta p^o = \frac{p - p_*}{\rho g_*^2/2} = 0.0674\, U_*^2 (1 - U^{o2}) \qquad (2.2)$$

This formula enables us to find the distribution $\Delta p^o(x^o)$ since the relationship $U^o(x^o)$ is known and is determined by Eq. (2.1).

The change in the value of U^o in a cylindrical channel with a different pattern* of velocity profile in the initial section characteristic of excess velocity U_* is illustrated in Fig. 5. It is evident that the greater the initial nonuniformity, the more intense is its equalization also, during which in accordance with Formula (2.2) a greater increase in pressure also occurs.

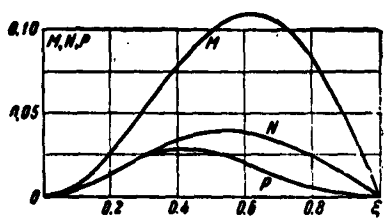

Fig. 4. Change in the coefficients M(ξ), N(ξ), and P(ξ) in cross-section of stream.

Case 2. A channel with variable cross-section and a straight-line generatrix ($R^o = 1 + \alpha x^o$). The differential Eq. (1.11) in the case in question takes the form

$$(1 + \alpha x^o)(1 + aU)\frac{dU}{dx^o} = 4\alpha U + (2\alpha c \mp k)U^2$$

The solution of this equation with the boundary condition of $x^o = 0$ and $U = U_*$ is expressed by the following simple formula:

$$\left(\frac{U}{U_*}\right)^{\frac{1}{4}}\left(\frac{1+mU}{1+mU_*}\right)^n = 1 + \alpha x^o \qquad (2.3)$$

Here are introduced the following designations

$$m = \frac{c}{2} \mp \frac{k}{4\alpha}, \qquad n = \frac{1}{4}\left(\frac{a}{m} - 1\right)$$

If, as in the preceding case, we introduce the relative axial velocity U^o relationship (2.3) will be rewritten thus:

---

* The nonuniformity of the velocity profile may be characterized by the coefficient $\nu = 1 - \tau$, where the value of τ is the ratio of the velocity averaged for flow rate to the velocity averaged for area

$$\tau = 2R^2 \int_0^R u^2 y\, dy \Big/ \left(2\int_0^R uy\,dy\right)^2.$$

Adopting Formula (1.2) for the distribution of the velocity we can show that $\tau = 1 + 0.0674\ U^2$. Thus the coefficient of nonuniformity $\nu = 0.0674\ U^2$ proves to be immediately connected with the excess axial velocity U.

$$(U^{\circ})^{\frac{1}{4}}\left(\frac{1+mU_{*}U^{\circ}}{1+mU_{*}}\right)^{n}=1+\alpha x^{\circ} \tag{2.4}$$

As we see, the change in velocity along the axis of the channel is determined by two parameters, U_* and α, characterizing the initial pattern of the velocity profile and the geometric properties of the channel, respectively.

Knowing the relationship $U^{o}(x^{o})$ we may find the distribution of the static pressure along the channel from Eq. (1.6). Omitting the intermediate calculations we will write the final expression relating the change in pressure, as in the preceding case, to velocity pressure $\rho g^2/2$ corresponding to the average (areal) stream velocity in cross-section $x^{o} = 0$.

$$\Delta p^{\circ}=(1+s+t)-\left(\frac{1+mU_{*}}{1+mU_{*}U^{\circ}}\right)^{4n}\left(\frac{1}{U^{\circ}}+s+tU^{\circ}\right) \tag{2.5}$$

Here

$$s=\frac{0.1348\,U_{*}}{m(1-4n)},\qquad t=\frac{0.0674\,(3-4n)\,U_{*}^{2}}{1-4n}$$

The evolution of the nonuniformity of the velocity profile in channels of the effuser and diffuser type is shown in Fig. 5 in the form of the relationship $U^{o}(x^{o})$ for three values (+0.05, -0.05, -0.1) for parameter α corresponding to central angles of cones of $5^{o}40'$ and $11^{o}30'$. As should have been expected the most rapid damping of the nonuniformity occurs in the effuser, in which process the effect of parameter U_* becomes weaker with the increase in the degree of constriction of the channel, and, for example when $\alpha = -0.10$ the curves of $U^{o}(x^{o})$ for $U_* = 0.5$ and 1.0 practically coalesce into one.

The change in the nonuniformity of the velocity profile along the channel occurs under the influence of two factors: turbulent mixing (proportional to the cross-sectional gradient of the velocity)

always leading to the leveling of the nonuniformity, and unequal acceleration of the fluid in different flow tubes on changing the area of the cross-section (with the same pressure change the slower jets accelerate in the effuser or slow down in the diffuser more than do the faster jets). The first of these factors in its purest form appears in the examples examined in Fig. 5 in the case of a cylindrical channel ($\alpha = 0$), and the second factor begins to prevail most powerfully over the first in the rapid area change of the cross-section, that is, in a large gradient of pressure (this is observed in the examples in Fig. 5, e.g., for the effuser even when $\alpha = -0.10$). In the flow in the effuser both factors act in the same direction and in the flow in the diffuser the second factor, in distinction from the first, favors the increase of velocity-profile nonuniformity. The common influence of the noted factors is very clear in the cases of flow shown in Fig. 5 in a weak diffuser (central angle $5^{\circ}40'$). At a greater initial nonuniformity ($U_* = 1.0$) both factors are approximately the same in strength: the curve of $U^{\circ}(x^{\circ})$ diverges weakly from the line of $U^{\circ} = 1$ (in the direction of $U < 1$); but at a smaller nonuniformity ($U_* = 0.5$) the first of the mentioned factors becomes considerably weaker than the second and as a result of the unfavorable effect of the second factor the nonuniformity of the velocity profile proves to be progressive downstream ($U^{\circ} > 1$).

In flow calculations in conical diffuser channels we should keep in mind the possibility of the stream breaking away from the wall of the channel when the size of the central angle exceeds 8 to 10°. The calculation method proposed here in the case of break-away diffusers may be applied only in the flow region where the zone of reverse currents has already disappeared and the velocity profile of form (1.1) has become autosimulating.

3. Comparison with the experimental data. Research on flow with a jet profile of velocity in a channel of cylindrical shape has been conducted by several authors. In particular, data of interest to us are contained in works by A. Ya. Cherkez [2] and also by W. Richards and W. Osborne [6] on low-pressure air ejectors and in research by V. M. Papin on a water ejector [7]. Figure 6 shows the experimental results of these authors together with the $U^o(x^o)$ curve computed of the theoretical relationship (2.1). In doing this, in each case we selected as a beginning for the calculation on the x-axis the cross-section in which $U_* = 1$; we took the value of the empirical coefficient $\underline{k}$ to be 0.26. As we see, the calculation method proposed here leads to completely satisfactory agreement with the experimental data on hand. The constancy of the empirical coefficient in the calculation formula under different experimental conditions which was discovered from a comparison of the theory with experiment should be especially emphasized.

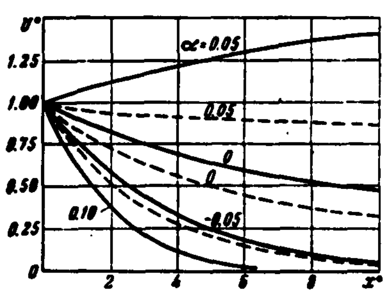

Fig. 5. Calculated relationship $U^o(x^o)$ for diffuser ($\alpha = 0.05$), cylindrical tube ($\alpha = 0$), and effuser channel ($\alpha =$ $= -0.05, -0.10$). Solid curves for $U_* = 0.5$; dashed, for $U_* = 1.0$.

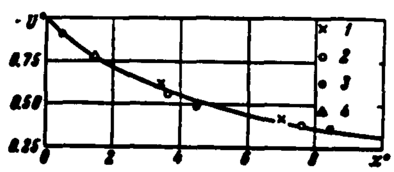

Fig. 6. Comparison of calculated curve of damping of relative axial velocity U^o with experimental data of various authors. Curve from Formula (2.1); points from experiments of 1) Cherkez [2], 2 and 3) Richards and Osborne [6], 4) Papin [7].

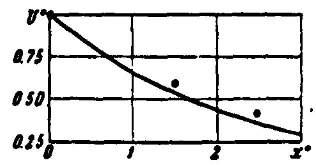

Fig. 7. Change in axial velocity $U^o(x^o)$ of nonuniform stream in effuser with straight-line generatrix ($\alpha = -0.0721$). Curve from calculation, points from author's experiments.

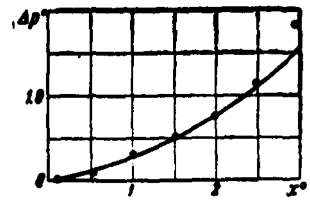

Fig. 8. Change in static pressure $\Delta p^o(x^o)$ in nonuniform stream in effuser with straight-line generatrix ($\alpha = -0.0721$). Curve from calculation, points from author's experiments.

It was of great interest to compare the theoretical and experimental results in the case of a channel with variable cross-section. Such experiments were conducted by the author on a model of a effuser mixing chamber which represented a truncated cone with an angle of $8^o20'$ ($\alpha = -0.0721$) 1040 mm long and with a cross-sectional diameter of 100 mm. Two coaxial streams of air, the flow rates per second of which could be independently regulated, were supplied to the entry section of the mixing chamber; this allowed us to create different nonuniformities of velocity profile in the entry section of the channel. Figure 7 shows the data of one of these experiments in the form of relationship $U^o(x^o)$; the calculation curve from Formula (2.4) is also shown here. The nonuniformity of the stream in the section taken to be initial was characterized by a value of $U_* = 1.0$. The empirical coefficient k was taken in the case examined to be 0.26, i.e., equal to the value derived for the flow in a cylindrical tube.

Figure 8 shows the experimental data obtained in the author's experiments on the distribution of static pressure along the wall of the channel; here is also entered the curve defined by relationships (2.4) and (2.5). From an analysis of Figs. 7 and 8 it follows that the calculation method proposed here is completely suitable for

practical use in studying the motion of a fluid with a nonuniform velocity profile of the jet type in a effuser channel with a straight-line generatrix.

REFERENCES

1. G. N. Abramovich. Applied Gas Dynamics. Gostekhizdat, 1953.

2. G. N. Abramovich. Theory of Turbulent Streams. Fizmatgiz, 1960.

3. K. Viktorin. Untersuchung turbulenter Mischvorgänge. Forsch. Ing.-wes., Bd. 12. S. 16-30, 1941.

4. S. Gol'dshteyn, ed. The Present State of the Hydro-Aerodynamics of a Viscous Fluid, Vol. 1. IL, 1948. [Translation of Modern Developments in Fluid Dynamics, Vols. 1 and 2. Oxford University Press, 1938].

5. E. Kamke. Handbook of Ordinary Differential Equations. Fizmatgiz, 1961.

6. W. G. Richards, W. C. Osborne. The very low pressure ejector. J. Inst. Heat and Ventilat. Engrs., Vol. 27, pp. 172-179, 1959.

7. V. M. Papin. Experimental Investigation of Phenomena Occurring in the Mixing Chamber of a Water-Jet Pump. DAN SSSR, Vol. 84, No. 5, 1952.

Received April 13, 1961.

DISTRIBUTION LIST

DEPARTMENT OF DEFENSE	Nr. Copies	MAJOR AIR COMMANDS	Nr. Copies
		AFSC	
		SCFTR	1
		ASTIA	25
HEADQUARTERS USAF		TD-Bla	5
		TD-Blb	3
AFCIN-3D2	1	AEDC (AEY)	1
ARL (ARB)	1	SSD (SSF)	2
		BSD (BSF)	1
		AFFTC (FTY)	1
		AFSWC (SWF)	1
OTHER AGENCIES			
CIA	1		
NSA	6		
AID	2		
OTS	2		
AEC	2		
PWS	1		
NASA	1		
RAND	1		